Sec. 12.2 Systems of Equations Containing Three Variables

To Obtain an Equivalent System of Equations –

1. Interchange any two equations in the system.
2. Multiply (or divide) each side of an equation by the same nonzero constant.
3. Replace any equation in the system by the sum (or difference) of that equation and a nonzero multiple of any other equation in the system.

To Solve with 3 Variables –

1. Use either substitution or elimination to combine two of the three equations.
2. Simplify.
3. Use another substitution or elimination to solve for one variable.
4. Now use that variable to find the other 2 variables (work in reverse order).

Ex: Solve the following linear equations.

$$\begin{cases} x+y-z=-1 \\ 4x-3y+2z=16 \\ 2x-2y-3z=5 \end{cases}$$

1+2: $3x+3y-3z=-3$
$+\ 4x-3y+2z=16$
$(7x-z=13)\times -5$

1+3: $2x+2y-2z=-2$
$+\ 2x-2y-3z=5$
$4x-5z=3$
$-35x+5z=-65$
$-31x=-62$
$\boxed{x=2}$

$7(2)-z=13$
$14-z=13$
$-z=-1$
$\boxed{z=1}$

$2+y-1=-1$
$y+1=-1$
$\boxed{y=-2}$

$\boxed{(2,-2,1)}$

Ex: Solve the following linear equations.

$$\begin{cases} 2x+y-z=-2 \\ x+2y-z=-9 \\ x-4y+z=1 \end{cases}$$

1+3: $2x+y-z=-2$
$x-4y+z=1$
$(3x-3y=-1)\times 2$

2+3: $x+2y-z=-9$
$x-4y+z=1$
$(2x-2y=-8)\times -3$

$6x-6y=-2$
$-6x+6y=24$
$0=22$

$\boxed{\text{No Solution - Inconsistent}}$

Ex: Find real numbers a, b, and c so that the graph of the function $y = ax^2 + bx + c$ contains the points (-1, 4) (2, 3) and (0, 1).

CREATE ONE EQUATION FOR EACH POINT - SOLVE FOR a, b, c.

$(-1,4) \rightarrow 4 = a(-1)^2 + b(-1) + c$

$(2,3) \rightarrow 3 = a(2)^2 + b(2) + c$

$(0,1) \rightarrow 1 = a(0)^2 + b(0) + c$

$4 = a - b + c$

$3 = 4a + 2b + c$

$1 = c$

$4 = a - b + 1$

$3 = 4a + 2b + 1$

$3 = a - b \longrightarrow 3 + b = a$

$2 = 4a + 2b$

$2 = 4(3+b) + 2b$

$2 = 12 + 4b + 2b$

$2 = 12 + 6b$

$-10 = 6b$

$-\frac{5}{3} = b$

$a = 3 + b$

$a = 3 - \frac{5}{3}$

$a = \frac{9}{3} - \frac{5}{3}$

$a = \frac{4}{3}$

$y = \frac{4}{3}x^2 - \frac{5}{3}x + 1$

Ex: Kelly has $20,000 to invest. As her financial planner, your recommend that she diversify into three investments: Treasury bills that yield 5% simple interest, Treasury bonds that yield 7% simple interest, and corporate bonds that yield 10% simple interest. Kelly wishes to earn $1390 per year in income. Also, Kelly wants her investment in Treasury bills to be $3000 more than her investment in corporate bonds. How much money should Kelly place in each investment?

x = amount in Treasury bills

y = amount in Treasury bonds

z = amount in coorporate bonds

$x + y + z = 20{,}000$

$.05x + .07y + .1z = 1390$

$x = z + 3000$

$z + 3000 + y + z = 20{,}000$

$.05(z + 3000) + .07y + .1z = 1390$

$2z + 3000 + y = 20{,}000$

$.05z + 150 + .07y + .1z = 1390$

$y + 2z = 17{,}000$

$.07y + .15z + 150 = 1390$

$y = 17{,}000 - 2z$

$.07y + .15z = 1240$

$.07(17{,}000 - 2z) + .15z = 1240$

$1190 - .14z + .15z = 1240$

$1190 + .01z = 1240$

$.01z = 50$

$z = \$5000$

$x = z + 3000$

$x = 5000 + 300$

$x = \$8000$

$y = 17{,}000 - 2z$

$y = 17{,}000 - 2(5000)$

$= 17{,}000 - 10{,}000$

$y = \$7000$

HW: pg 755 #5,8,13,18,21-23*,25**

*Set up system of equations but do not solve.

**Answer first question only.